DEPOLARIZING MATHEMATICS AND RELIGION
by Sarah Voss

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A Calculating Myth

In the beginning there was Nothing.
Nothing grew bored with being Nothing
and decided to become Something.
Something wanted to discover Everything,
so Something split into Parts
and the Parts, fearing they would forget
how they fit into Something, sought out Order.
Order gave the Parts Numbers, which the Parts
gathered unto themselves in beautiful Proportions.
Then, out of Nowhere (closely associated with
Nothing) came Irrationality.
Irrationality declared flatly that the Parts
were really Nothing after all. Enlightened,
the Parts quietly slipped back into Something,
which they now recognized was really Nothing,
and left the discovery of Everything to Numbers.

For many people, mathematics and religion seem to exist on opposite ends of a linear scale. Peer at it one way and we see religion; look the other way and we find mathematics. And never the two shall meet. With one view we find ourselves latching on to such words as *analogy, myth, stories*. The other we describe with words such as *analysis, logic*, and *scientific investigation*. That mathematics and religion need not be so polarized, and in fact have not always been so rudely separated, is in large part the focus of this study.

We inherit a recent tradition which is rich in this polarization. In the early 1960's, for instance, one attempt to show how "the two disciplines can at certain points supplement each other" (Stuermann 11) degenerated into the supposition that "science performs one simple service for religion. It lays bare new sources of anxiety, evil, disaster, and peril" (154). More specifically,

Science deals with carefully quantified, reliable perceptions. It does not deal with what is ultimately real. By its own definition of fact, science can have nothing whatever to do with God or other supernatural entities and forces. It is important to recognize that science is clearly separated from metaphysics and theology by its own criteria, not merely by the declarations of religious persons (47).

From the other side of the fence, we find mathematician E.T. Bell citing a "man 0' science"! in his 1930's *Search for Troth* on the subject of religiosity:

Those who seek support for their dogmas are more concerned with gaining a powerful support for their own prestige and the security of their worldly livings than they are in maintaining and spreading what they are popularly supposed to believe is the truth. Their faith is a depression. As a pendant to this, those men [or women2] of science who permit themselves to be used as catspaws in this shady transaction cease, *ipso facto*, to be scientists and their support, whatever may be its value as the sincere expression of the convictions of devout men, has no scientific value at all. (Bell 195-6)

Within my own experience I have encountered similarly abrupt dismissals of any attempt to integrate these two extremes. One learned theologian, for instance, insisted I was comparing oranges with fork-lifts, and, rebuking my suggestion that both mathematics and religion are symbol systems, delivered the following litany:

Math is not a social system or a social institution. Math is not a system but a set
Math is not symbolic but significant
Math is not factual but mental.
Math is not mood-filled but non-emotional.
There are no symbols in math other than math itself.

Then, as though to emphasize the depth of my misperception, he pointed out that "If you want to understand when and how math functions, trace the grades of Japanese students in U.S. colleges. The more math, the higher their grades - until they get into the humanities where passing is really difficult"

I admit I was taken aback by his comments, particularly since, as a mathematics professor, I like to think I already have a fair understanding of when and how mathematics functions. Still there is value in reporting his response, for it underscores the extent to which we humans find it difficult to accommodate apparently opposing viewpoints. Such polarization, unfortunately, perpetuates a myth that blinds us to rich possibilities of integration.

Cultural anthropologist Clifford Geertz has offered the following definition of religion:

Religion is a system of symbols which acts to establish powerful, pervasive and long lasting moods and motivations in people by formulating conceptions of a general order of existence and clothing these conceptions with such an aura of factuality that the moods and motivations seem uniquely realistic. (Geertz 91)

This definition has been offered by the theological community as an appropriate answer to the question "What is religion?": for the purposes of this study, I accept its validity. With a little word play, it is possible to alter Geertz' wording and arrive at several other meaningful definitions. For example, by replacing the phrase "formulating conceptions of general order of existence" with "formulating conceptions of collective and legislative action," the concept of "government" is nicely addressed. Likewise, by substituting for that same phrase the words "formulating concepts of provisions of society," we arrive at a workable definition of "economics," and so on.3 In a similar vein, we might replace the single word "religion" with the one word "mathematics" and arrive at the following possibility:

Mathematics is a system of symbols which acts to establish powerful, pervasive and long lasting moods and motivations in people by formulating conceptions of a general order of existence and clothing these conceptions with such an aura of factuality that the moods and motivations seem uniquely realistic.

Assuming, momentarily, the validity of this new definition, it is important to consider the implications which follow from this interchange. In particular, the replacement of "religion" with "mathematics" does not necessarily imply the equivalence of the two terms. Given that "A is B" and "e is B," we cannot conclude that "A is e" "Sarah is a woman" and "Elizabeth is a woman" does not mean that "Sarah is Elizabeth." It does, however, suggest a strong and undeniable connection between the two "symbols" thus interchanged - a connection rooted in similarity rather than in polarization.

But just how valid is this altered version? When Geertz originally offered his definition, he distinguished five separate parts, each of which he discussed in some detail. Similarly, there are five distinct aspects to this altered version. Mathematics is (1) a system of symbols which acts (2) to establish powerful, pervasive and long lasting moods and motivations in people by (3) formulating conceptions of a general order of existence and (4) clothing these conceptions with such an aura of factuality that (5) the moods and motivations seem uniquely realistic. Following a vein of reasoning similar (at least in structure) to that of Geertz, I will examine each of these five aspects with an intent to justify its appropriateness to mathematics.

...a system of symbols which acts to...

Declaring mathematics "a system of symbols" seems to me (the mathematician) so obvious that it is almost trite. Mathematics abounds in symbols, most of whose meanings have been carefully considered and cross-culturally agreed upon. Furthermore, this system of symbols represents the epitome of economy and efficiency in human expression. The integral notation developed in the 17th century by Leibniz is only one example of how an entire, complicated process is captured precisely in a few swift pen strokes.

However, as the almost ritualistic litany cited above suggests, the symbolic nature of mathematics may not be as obvious to others as it is to me. Theologians frequently differentiate between "symbol" and "sign," with the latter being representative of a known content and the former representing a content while also transcending that content in some way that cannot be expressed in rational terms. This subtlety is aptly illustrated by the German word for symbol, *Sinnbild*: the *Sinn*, or meaning, refers to the conscious, rational sphere, whereas the *Bild*, or image, belongs to the irrational sphere, the unconscious (Jacobi 96). A stop light, for instance, is a "sign,"but a cross is a "symbol" - at least for some cultures.

The apparent afterthought of the preceding sentence deserves emphasis, for, as Geertz himself notes, "symbol" often refers to a great variety of things, and sometimes to a number of them all at the same time (Geertz 91). Rather like "beauty," "symbol" may well be in the eye of the beholder. Geertz speaks of "culture patterns" as symbols or complexes of symbols, and insists that they "have an intrinsic double aspect: they give meaning, that is objective conceptual form, to social and psychological reality both by shaping themselves to it and by shaping it to themselves" (Geertz 93). Likewise, to the degree to which mathematics offers us symbols which in some way transcend themselves, that is, stand for a content while simultaneously shaping that content, then mathematics may be appropriately described as a symbol system. Recent avenues of study such as those explored by sociologist Sal Restivo put forth a strong if not indisputable argument for precisely such a sociology of mathematics (See Restivo, part 2).

Mathematics offers us some of humankind's oldest symbols. Names for certain numbers are among the oldest "known" words. Buckminster Fuller, himself an unusual integrator of mathematics with other disciplines, pointed out in his prose-poem "Numerology" that if we were to collect all the various names for numbers from all the various world languages, we would be surprised at how many we would recognize without difficulty. Comparing the names for just the first two cardinal numbers ("one" and "two"), we would observe that in nearly every language the symbol for "one" starts with a vowel and has vowel sound emphasis, whereas the "two" has a beginning consonant sound and a consonant sound emphasis. Fuller concludes (with a touch of mythology) that number names grew from the same fundamental roots:

We either have to say that some angels
Invented the names for numbers
And the phonetically soundable
Alphabetical letter symbols
With which to spell them
And wrote them on parchments
And air-dropped those number-name leaflets
All around the spherical world,
Thus teaching world-around people the same number names;
Or we have to say that numbers were invented
By one-world-around-traveling people.
(Fuller 739-41)

Fuller attributes the formulation of numbers to sailors, specifically the Polynesians, who "inculcated their use all around the world (741)," Calling them navigator priests ("the only people who knew that the Earth is spherical, that the Earth is a closed system with its myriad resources chartable") (751), he traces their path westward through Malaysia and southern India, across the Indian Ocean to Mesopotamia and Egypt and thus into the Mediterranean. Most (Western) histories of mathematics begin their serious study with the essentially practical reckoning systems4 of the ancient Babylonian, Egyptian, and Cretian cultures; Fuller believes it is likely that the powerful priest-mathematicians of these societies were really "the progeny of mathematician navigators of the Pacific come up upon the land" (750).

The way mathematics developed as a system of symbols is certainly linked to the way in which language developed as a system of symbols. But even more importantly, our mathematical system has changed (is changing) in accordance with the development of human conscious thought. Humankind moved from chanted symbols to spoken numbers to finger tallying to incised bone artifacts to clay tablets, and so on. It also moved from the ability to associate one object with an abstract scratch on a stone to the capacity for recognizing a group of objects as the concept "five" to the ability to detach the number sequence totally from the objects being counted, and so on (Burton 2). Inasmuch as religion is also a system of symbols acting by means "of formulating conceptions," all of which may be analyzed at various levels of abstraction, it is probable that the two systems share commonalities just by virtue of their apparent embeddedness in human thought.

...to establish powerful, pervasive, and longlasting moods and motivation in people...

That the system of symbols called mathematics acts "to establish powerful, pervasive and longlasting moods and motivations in people" may be verified by an excursion deeper into these history books. Consider, for example, the man who became so preoccupied with the decimal calculation of pi that he devoted his entire life to this one pursuit, finally achieving a rendition (35 decimal places) which can now be far surpassed in only moments by computers. Or consider the motivating factors which initiated and perpetuated the bitter rivalry between Newton and Leibniz over their essentially "simultaneous" discovery of the calculus. Or those which caused Gauss and Bolyai to refrain in the 1800's from publishing their separate discoveries of non-Euclidean geometry. On a more "religious" note, what about the member of the Pythagorean mystic brotherhood who was allegedly thrown overboard at sea by his peers because he discovered that not all numbers are commensurable? Then there was the turmoil and punishment Galileo suffered for his refusal to deny the truths his mathematical symbols told him. Such examples, and many others like them, justify the appropriateness of applying the second part of Geertz's definition to "mathematics" without loss of meaning.

...clothing these conceptions with such an aura of factuality
that the moods and motivations seem uniquely realistic...

The last two parts of this same definition are, from my standpoint, more or less axiomatic givens insofar as they apply to mathematics. Who would deny that mathematics is a system of symbols which motivates powerful energies in people to formulate certain conceptions, and that (4) it clothes these conceptions "with such an aura of factuality that" (5) "the moods and motivations seem uniquely realistic?" Certainly none of those mathematicians mentioned in the previous paragraph. The apparent human need for consistency, for finding order in the midst of chaos, ensures that no mathematician will willingly negate the factuality and realism of that which embodies the core of his or her belief system, especially not when these beliefs are accompanied by powerful, pervasive and long-lasting moods and motivations. The seeming uniqueness of these moods and motivations is more questionable (at least to the mathematician.. who is used to working with changeable axiomatic propositions) but probably no less intuitively feelable than it is to any other member of the human species. To be sure, I am not asserting the truth or falsity of the certain conceptions themselves; rather, I am agreeing to the likelihood of our readiness to perceive them as valid. The non-mathematician or non-scientist might be even more eager to grant this appearance of factuality to mathematics than would mathematicians themselves. Mathematics, after all, is popularly considered an "exact" science, the epitome of logic, the one place where you can arrive at right or wrong answers with some assurance of accuracy. Who but a mathematician would suggest that two plus two is not four?6

...formulating conceptions of a general order of existence...

It remains to consider the third part of our altered definition: is mathematics, then, a system of symbols which formulates conceptions **of a general order of existence?** The answer is a clear and unambiguous yes and no. It is "no" in the same sense of practicality that permeated the mathematics of ancient Western cultures. Mathematics is and always has been what Olaf Pederson, in a collection of essays on physics, philosophy, and theology commissioned by the Vatican City State, called "a vehicle of description" and "a tool of discovery" (Pederson 132). Throughout the ages, such description and discovery has been used to alter, improve, expand, predict, categorize, etc., the relationship between humans and their environment. Sometimes mathematics has been misused as a vehicle of description and a tool of discovery: consider, for example, how often history portrays mathematics as the handmaiden of power-hungry priests who hoarded learning in their own self-interest (Burton 91). Still, mathematics is "not just another scientific language to be used or rejected as one please[s]. It [is] much more potent than ordinary language, and the only one...able to produce a fascinating result" (Pederson 132).

This is the action-oriented aspect of mathematics, and, in a sense, it might be likened to the rituals, the daily worship, the everyday, common interests of religion. These aspects, mathematical *or* religious, allow humankind to go about the business of daily living. These are "do it" aspects, not "what's it all about" aspects. It is in this sense that mathematics is *not* a system of symbols which attempts to formulate conceptions of a general order of existence. It is in this same sense that religion, also, is not an attempt to formulate such conceptions.

But, just as the ancient Egyptians used their practical mathematics to help prepare them for a "spiritual" afterlife (see Note 4), the distinction between the practical and the philosophical is a slippery one. A careful exploration of the role of mathematics in the value-oriented world of humankind shows that, time after time, mathematical pursuits intertwined with religious preoccupations.

"From the very beginnings of the development of scientific thinking in the ancient world, first in Babylon and then in Greece, thought about God or the gods -*theology* - and thought about the world *natural science* - were so intermingled that often it was quite impossible to differentiate between the two... It was primarily for religious reasons that people from the unnamed Babylonians through the pre-Socratics, the Pythagoreans, Plato, and Aristotle attempted to understand the heavens, to trace their geometry and calculate their ratios" (Nebelsick xiv).

When viewed from this perspective mathematics does, indeed, act to formulate conceptions of a general order of existence.

If out of the mouths of Buckminster Fuller's Polynesian sailors came chanted numbers, it would seem that they were to become "enchanted" numbers for much of the world. To the Pythagoreans, for instance, numbers were not "the means of calculating the relationships between the things of reality, *they were reality*" (Nebelsick 14). In China, divination, which was "an attempt to ascertain truth on a level other than that of verifiable analysis or quantifiable proof, and by means other than those which depend on reason" (Loewe 39), was almost totally dependent upon a numerical system of throwing stalks in a pattern of 64 possible hexagrams. In Tibet, where, divination was intimately though not exclusively connected to number (primarily via dice throwing), "it was never really possible to separate the religious and the secular" (Radha 7). In India, numbers are "of the kind of Brahma"; numerical allegory found in the Jewish Qabbalah is perhaps in its most developed form; number appears in Islamic mystical thought; Augustine found numbers in the scriptures to be both sacred and mysterious (Schimmel 7). Johannes Kepler, Galileo Galilei, and Isaac Newton cast horoscopes as well as mathematical theorems, and, indeed, the entire Renaissance was "a world in which alchemy was the search for divine essence which was thought to be the basis and the unification of all material reality" (Nebelsick 214).

Some would (do) argue that humankind's attempt to discern an understanding of the All through mathematical insight ended with the Copernican revolution and the birth of "modern" science.

But, as Annemarie Schimmel put it, "the mathematical spirit is innate in man and manifests itself wherever human beings live" (Schimmel 13). Chances are7, we of more recent times have simply adopted a more probabilistic approach to formulating conceptions about the general order of existence.

For one thing, mathematics permeates far more of the texture of today's life than is generally recognized. Psycho-social statistics is big business, statistical sampling and polling impact our commercial and political realms, economic theory can no longer be understood without a solid background in mathematics, the life sciences of biology and medicine are increasingly mathematical, and even linguistics is more about mathematical-like languages than about dictionary-like compilations (Davis 10). It is, of course, broadly possible to lump the significance of all this into the category of practical mathematics, to point an unforgiving finger at the evils science and technology have wreaked upon society, and. to cry out that modern day science/mathematics has only severed the threads which link us in any way with God, religion, or the understanding of our "true" existence.

To do this, however, is to ignore the possibility that the world may be essentially mathematical in nature, in which case understanding mathematics is an obvious first step to understanding the world. A large body of theoretical mathematics is increasingly influencing our *popular* philosophical literature with precisely that intent: consider, for instance, such recent bestsellers as James Gleick's *Chaos*, Douglas *Hofstadler's Godel, Escher, and Bach*, and Stephen Hawking's *A Brief History of Time*, all of which are representative of a decided trend to explain the universe in terms of mathematical concepts. In *Descartes' Dream: The World According to Mathematics*, by Davis and Hersh, we find a surprising echo of the Pythagorean claim the" All is number":

"God is a Mathematician" is a modern formulation meaning that the way of the world is mathematical, that mathematics provides the key to the universe, that God, as the Prime Mathematician, set up the universe according to the principles of mathematics. This view may be slightly egocentric, perhaps, and not necessarily subscribed to by theologians. It is a view that is widely held today by physicists (who mayor may not use the word "God") in order to answer the unanswerable question of why mathematics is such an effective tool in theoretical physics. It is the view which lies behind a great deal of the recent mathematizations of a variety of disciplines, history, sociology, psychology. The world is mathematical, and hence, to interpret it properly, one must use mathematics (Davis 233).

And Rudy Rucker, whose 1987 book *Mind Tools* deals with the mathematics of information processing, puts it into even more modern lingo when he defines reality as "an incomprehensible computation by a fractal CA of inconceivable dimensions" (Rucker 314). If this is nothing else, it is surely "formulating conceptions of general order of existence" which are mathematical rather than religious in nature.

Or, are they really religious as well? Herein lies once again the question of what happens if the word "mathematics" is substituted for the word "religion" in Geertz' definition without loss of meaningfulness. Geertz himself has stated elsewhere (Geertz, Myth viii) that "it is when two (or more) scholars realize that, for all the differences between them, they are attacking highly similar issues, trying to solve closely related puzzles, that communication between them begins to look like a practical policy rather than an academic piety." One such commonality of vision is what Geertz calls "the systematic study of meaningful forms."

One such meaningful form particularly relevant to this study is the Moebius strip analogy found in Paul van Geert's discussion of the mind-world dichotomy. The strip is reproduced below, along with a brief excerpt from van Geert's commentary.

IMAGE

Figure 13. 1 A geometrical Analogue of the dualistic and the monistic model of the mind-world distinction.. In (b), the mind is represented by the inner, whereas the world is represented b)' the outer, domain or a closed curve. It is impossible to relate all internal element (a concept, image. etc.) with an external element (a referent an object. etc.) without crossing the boundary between mind and world. In (c). the curve is drawn onto a Moebius-plane (see (a)). If one follows the dotted line, it is possible to connect internal and external elements without ever crossing the curved line (although this seems illogical from a two-dimensional point of view)

The Moebius strip provides an ideal illustration of one mathematical symbol which may act "to establish powerful, pervasive and long-lasting moods and motivations in people by formulating conceptions of a general order of existence...": hence it affords an excellent way in which to draw this study to a close. Although it is a sophisticated concept originating from the specialized area of mathematics known as topology, it lends itself well to a simple geometric interpretation. About a century ago, August Moebius discovered that a strip of paper, if given a single twist and secured together at the ends, forms what is now known popularly as the Moebius strip (Smith, 402).

The intrigue of this strip is that it has only one side. School children are usually fascinated when they first encounter a Moebius demonstration, for it provides a concrete example of something which is directly counter to their intuition; so, too, does, for example, the cross, a symbol of the Christian religion. The Moebius symbol is powerful enough to have captured the imagination (and motivation) of the Dutch artist, Escher, who transformed it into a work of art, perhaps as meaningful to some as Michelangelo's beautiful paintings on the ceiling of the Sistine Chapel. Rucker's use of it to demonstrate a "general order of existence," i.e., the mind-world dichotomy, is not so very different from the bread/body, wine/blood symbols of Christianity. And my use of it now to provide a visual image suitable to resolve the polarization between mathematics and religion will, I hope, appear to some as both factual and realistic. When the two poles of religion and mathematics are likened to "opposite" sides of this little strip of paper representative of a highly sophisticated human endeavor, it is immediately apparent that the polarization of mathematics and religion is, as the mystics might say, but an illusion.

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NOTES

1. Not Bell, but in keeping with the general spirit of his remarks.

2. My emphasis.

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RadhalRinpoche, Lama Chime, "Tibet",

3. With thanks to Bruce Malina, from whose lecture I arrived at this insight.

4. "Practical" here takes on new connotations, e.g., one of the earliest recorded sets of very large numbers may be found in the Egyptian Book of the Dead, a collection of religious and magical texts who main aim was to secure a satisfying afterlife for the deceased. (Burton 11)

5. Ludolph van Ceulen of Germany used polygons having 2 sides to arrive at his figures. His achievement was considered so exceptional that the number was engraved on his tombstone. (Eves 93).

6. Paul van Geert, for one, addresses the operational fallacies of such a statement in his 1983 study of perception, cognition, and language. The typical argument involves an overlap of Venn diagrams as in the situation where a man and woman, each already the parent of one child, marry and together become parents of another child. Thus, each parent has 2 children, yet 2 plus 2 clearly do not equal 4.

7. Pun intended.

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